



# Modelling urban development in New Zealand

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Stuart Donovan, Arthur Grimes,  
David C Maré

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BUILDING BETTER  
HOMES, TOWNS  
AND CITIES

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whakamāhorahora

## Contact details for main authors

Stuart Donovan  
Department of Spatial Economics  
Vrije Universiteit Amsterdam  
E-mail: s.b.donovan@vu.nl

Arthur Grimes  
Motu Economic and Public Policy Research, and  
Victoria University of Wellington  
arthur.grimes@motu.org.nz

David C Maré  
Motu Economic and Public Policy Research Trust  
dave.mare@motu.org.nz

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## Disclaimer

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PO Box 24390    [info@motu.org.nz](mailto:info@motu.org.nz)    +64 4 9394250  
Wellington    [www.motu.org.nz](http://www.motu.org.nz)  
New Zealand

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## Abstract

We formulate and estimate a simple dynamic spatial general equilibrium model of urban development. Notwithstanding its simplicity, the model allows for adjustment frictions in housing markets; workers with heterogeneous productivities and preferences; and agglomeration economies in production and consumption. We estimate our model as a system of equations using panel data for workers residing in 132 urban settlements in New Zealand for the period 1976 to 2013. In terms of housing markets, we find strong evidence of increasing marginal costs and large adjustment frictions. The latter suggests demand shocks lead to temporarily elevated prices. In terms of agglomeration economies, we find New Zealand's cities and towns offer economies of scale to producers, in the form of higher wages, but diseconomies of scale to consumers. By exploiting the panel structure of our data, we consider whether our findings are stable over time. We use the results of our model to compare relative productivity and amenity levels in New Zealand's cities and towns and consider implications for research and policy.

## JEL codes

R11, R12, R23

## Keywords

urban development; location choice; dynamics; wages; rents; New Zealand

## Summary prose

*Over space and time  
The attributes and prices  
Of cities differ*

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# 1 Introduction

What factors influence urban development?<sup>1,2</sup> Intuitively, cities that are attractive can be expected to be larger and grow faster, and vice versa. The aim of this working paper is to understand some of the factors that affect urban development in New Zealand.

To do so, we formulate and estimate an economic model. Our model builds on a large body of spatial economic literature, much of which stems from original work by Roback (1982; 1988). In Roback’s model, locations are treated as bundles of *prices*—typically wages and rents—and *amenities*—such as climate and infrastructure. Together, prices and amenities define the attractiveness of locations to people and firms. Roback’s key insight was to invoke the so-called “spatial equilibrium” condition, in which households and firms are assumed to be indifferent between locations. In spatial equilibrium, prices and amenities adjust such that no household or firm can make themselves better off by changing their location.

Using the concept of spatial equilibrium, researchers have derived theoretically consistent models of location choice that provide useful empirical insight. Such models predict, for example, that households will accept lower wages and / or pay higher rents to reside in locations with high levels of amenity. Similarly, firms will accept higher costs—in the form of higher wages and rents—to operate in locations where there are productive advantages. These theoretical predictions have found empirical support. Gabriel and Rosenthal (2004) use Roback’s model to derive indicators of locational attractiveness that predict population growth in the U.S. In New Zealand, work by Donovan (2011), Grimes et al (2017), and Preston et al. (2018) use similar methods to reach similar findings. Other studies adapt Roback’s model to impute monetary values for unpriced local amenities, such as air quality (Bayer, Keohane, & Timmins, 2009) and commuting costs (Albouy & Lue, 2015). The theoretical consistency and empirical validity of Roback’s model has seen it become a workhorse model in applied spatial economics research, and one that we adapt and extend in this working paper.

Notwithstanding the merits of Roback’s model, large gaps remain in our understanding of location choice, some of which we address in this paper. Notably, the concept of spatial equilibrium seems to be incompatible with the existence of migration: If spatial equilibrium

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<sup>1</sup> While we refer to our units of observation as “cities”, many are much smaller than what is normally considered as a city. We also use the terms “urban settlements” and “locations” interchangeably with “cities”.

<sup>2</sup> Grimes et al. (2016) consider this issue using a dataset of 56 New Zealand towns from 1926 to 2006. They find land-use capability, sunshine hours, human capital, and proximity to Auckland are associated with (long-run) population growth. Here, we consider more settlements, albeit over a shorter period.

implies prices and amenities instantaneously and perfectly adjust to leave people indifferent between locations, then why do we observe steady flows of migrants both internationally and domestically? In this paper, we consider whether housing market frictions slow progress towards spatial equilibrium. To test our hypothesis, we formulate and estimate an exceptionally simple dynamic spatial general equilibrium model that links the location choices of households to firm's demand for labour and the supply of housing, where the latter is subject to adjustment frictions. Whereas much of the existing literature focuses on long-run cross-sectional outcomes between locations, this paper joins the small but growing body of studies that uses a dynamic model. We estimate our model using a unique panel of data for 132 urban areas in New Zealand covering eight censuses in the period 1976–2013.

We are not the first to use the Roback model to analyse spatial dynamics. Chen and Rosenthal (2008) derive a panel of quality of life and business measures from the 1970–2000 US Census and find households prefer warmer climates while firms prefer large cities. Over time, Chen and Rosenthal find cities with improving business environments attract more workers, especially the highly skilled, whereas cities with improving consumer amenities attract retirees. Partridge et al (2012) analyse domestic migration and find no evidence the U.S. is approaching a spatial equilibrium, possibly due to a combination of historical shocks and market frictions that lead to a permanent disequilibrium. Glaeser et al (2014) adapt the Roback model to a dynamic spatial general equilibrium setting, generating predictions for house prices that are broadly consistent with empirical data. More recently, Ganong and Shoag (2017) present evidence that high housing prices caused by land use regulations serve to stymie domestic migration. Our approach is closest to Glaeser et al (2014), although we (1) allow for agglomeration economies in production and consumption, (2) focus on housing rents rather than house prices, and (3) model adjustment frictions in regional housing markets.<sup>3</sup>

We have three main results. First, we find empirical evidence of agglomeration *economies* in production and *diseconomies* in consumption. This finding suggests the productive advantages of New Zealand's larger cities are, to some extent, offset by consumption disadvantages, such as congestion. Second, we find evidence urban housing markets are subject to Ricardian notions of comparative advantage, in the sense marginal housing costs rise with urban size—even when controlling for income. Intuitively, this implies low-cost housing options are developed first. Third, and perhaps more importantly, we find evidence of adjustment frictions in regional housing markets. These frictions attenuate the speed with which the urban population adjusts to changes in the prevailing levels of prices and amenities, such as what

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<sup>3</sup> In the Appendix, we also show how our model is readily extended to the case of heterogeneous workers.

might arise in response to a technological shock. Indeed, our estimated parameter values suggest the process of population adjustment in response to shocks manifests over several decades, which results in relatively persistent migration flows.

Using our model, we draw out several insights. First, we find few significant departures from spatial equilibrium in 2013; most cities and towns are close to where we would expect. Second, we consider the dual role of cities as places of production and consumption. On average, we find a negative association between these outcomes: That is, cities that are more advantageous for firms tend to be less advantageous for households, and vice versa. Third, we show variations in the utility of locations in 2006—as calculated from our model—predict subsequent population growth. Fourth, we observe that departures from spatial equilibrium appear to have increased in 2013 vis-à-vis 2006, which may reflect the lingering effects of shocks like the Global Financial Crisis and the Christchurch Earthquake. And finally, we find differences in the value that households attach to locations is partly explained by natural amenities, such as climate, geography, and recreational activities. Together, these amenities explain approximately one-quarter of the variation in the value of locations to households.

What are the limitations of our approach? There are several. First, as the Roback model treats cities as points, we do not model intra-urban outcomes, such as congestion. Instead, our model captures intra-urban effects only in aggregate, for example via city-level agglomeration economies. Second, we do not control for a host of factors, such as industrial composition and worker characteristics, which are endogenously determined with spatial economic outcomes. Omitting these characteristics means our estimates of agglomeration economies are likely to be biased.<sup>4</sup> Third, we assume most economic processes, for example those relating to agglomeration economies and housing markets, are identical across cities. This means that changes in population affects all cities similarly. This is uninformative and runs counter to evidence that local geography and policy, for example, affects the responsiveness of housing markets over time. Fourth, while we find evidence that local amenities are important in aggregate—and we identify a specific role for natural amenities, such as climate, geography, and recreation—we do not explain observed differences in the attractiveness of locations in ways that can directly inform policy. In the absence of causal links to policy, our results are more descriptive than prescriptive. Given these limitations, we finish this working paper with a discussion of the opportunities for further research.

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<sup>4</sup> In the Appendix, we extend our model to allow for heterogenous workers that differ in their productivities and preferences. While we do not estimate this model in the current paper, this is the focus on ongoing work.



## 2 Economic Model

Here, we formulate the economic model to guide our analyses. Whereas the basic Roback model is “static”—in the sense that spatial equilibrium holds between locations—we translate the model into a dynamic setting, where spatial equilibrium holds both *between* cities and *over* time. Moving to a dynamic setting allows us to make better use of the available data and incorporate adjustment frictions. The following sub-sections present preliminary aspects of the model before considering the firm, worker, and housing aspects of the model in turn.

### 2.1 Preliminaries

To begin, we assume identical workers choose in every period,  $t$ , where to live from  $C$  different cities, indexed by  $c = 1, \dots, C$ . Workers supply one unit of labour and earn a locally determined income ( $\check{w}_{c,t}$ ), which is spent on (1) housing ( $H_{c,t}$ ) and (2) consumption of  $Y_{c,t}$ . Each worker consumes one unit of housing at price,  $r_{c,t}$ , which is also determined locally. The price ( $p_{c,t}$ ) of the tradeable output ( $Q$ ) is constant across cities; we take  $p_{c,t} = 1$  as the numeraire. In each city, there exists a representative firm that uses commonly available technology to produce the tradeable good  $Q$  from two inputs: (mobile) labour,  $N_{c,t}$ , and (immobile) floor space,  $L_{c,t}$ , under constant returns to scale. We assume the price of commercial floor space is a fixed multiple  $\mu$  of the rental price of housing. The attractiveness of cities to workers varies with amenities and prices,  $\check{w}_{c,t}$  and  $r_{c,t}$ . In each city  $c$ , we distinguish between the wages *paid by firms*, denoted by  $w_{c,t}$ , and the wages *earned by workers*, denoted by  $\check{w}_{c,t}$ . This distinction allows for residents to commute to other locations for work purposes.

### 2.2 Firms

We assume firms use a Cobb-Douglas production function operating under constant returns to scale. The representative firm in city  $c$  then maximises the following profit function:

$$\pi_{c,t} = g(P_{c,t})L_{c,t}^\gamma N_{c,t}^{1-\gamma} - \mu r_{c,t}L_{c,t} - w_{c,t}N_{c,t} \quad \text{Eq. 1}$$

Where  $g(P_{c,t})$  denotes a Hicks neutral productivity shifter that is a function of local productive amenities.  $P_{c,t}$ , floor space  $L_{c,t}$ , and the number of workers  $N_{c,t}$ . Prices of floor space and wages in city  $c$  are denoted by  $\mu r_{c,t}$  and  $w_{c,t}$ , respectively, where  $\mu$  denotes a constant multiple of the price of residential housing.

Using the first-order conditions  $\frac{\partial \pi_{c,t}}{\partial L_{c,t}} = g(P_{c,t})\gamma L_{c,t}^{\gamma-1} N_{c,t}^{1-\gamma} - \mu r_{c,t} = 0$  and  $\frac{\partial \pi_{c,t}}{\partial N_{c,t}} = g(P_{c,t})(1-\gamma)L_{c,t}^\gamma N_{c,t}^{-\gamma} - w_c = 0$ , we derive an expression for the firm's demand for floor space  $L_{c,t}$  as a function of labour,  $N_{c,t}$ , and prices  $L_{c,t} = N_{c,t} \frac{\gamma}{(1-\gamma)} \frac{w_{c,t}}{\mu r_{c,t}}$ . Assuming free-entry of firms leads to zero-profits, we derive  $w_{c,t} = (1-\gamma)\gamma^{\frac{\gamma}{1-\gamma}} P_{c,t}^{\frac{1}{1-\gamma}} \left(\frac{1}{\mu r_{c,t}}\right)^{\frac{\gamma}{1-\gamma}} = \lambda g(P_{c,t})^{\frac{1}{1-\gamma}} \left(\frac{1}{\mu r_{c,t}}\right)^{\frac{\gamma}{1-\gamma}}$ . Taking logs yields the “wage”—or iso-cost—equation for each city,  $\log w_{c,t} = \lambda + \left(\frac{1}{1-\gamma}\right) \log g(P_{c,t}) - \left(\frac{\gamma}{1-\gamma}\right) \log r_{c,t}$ , where the constant  $\lambda = \log(1-\gamma) + \left(\frac{\gamma}{1-\gamma}\right) \log \gamma - \left(\frac{\gamma}{1-\gamma}\right) \log \mu$ .

We model agglomeration economies in production using the function  $g(P_{c,t}) = P_{c,t} E_{c,t}^\epsilon$ , where  $E_{c,t}$  denotes the “effective population” and is defined as follows

$$E_{c,t} = N_{c,t} + \sum_{j \neq c} \frac{N_{j,t}}{d_{j,c}} \quad \text{Eq. 2}$$

Effective population  $E_{c,t}$  captures both *own* and *other* city population effects, where the latter attenuates by  $d_{j,c}$ , the distance by road between cities  $c$  and  $j$ , which is constant over time.<sup>5</sup> Using this specification of amenities, the firm's “iso-cost” equation becomes:

$$\log w_{c,t} = \lambda + \left(\frac{1}{1-\gamma}\right) \log P_c + \epsilon \left(\frac{1}{1-\gamma}\right) \log E_{c,t} - \left(\frac{\gamma}{1-\gamma}\right) \log r_{c,t} \quad \text{Eq. 3}$$

Eq. 3 suggests wages increase with net productive amenities,  $\log P_c + \epsilon \log E_{c,t}$ , and decrease with rents,  $r_{c,t}$ , which is intuitive.

## 2.3 Workers

Mobile workers locate in the city  $c$  that maximises their utility  $U_{c,t}$ , which depends on the local amenities,  $f(A_{c,t})$ , and consumption,  $Y_{c,t}$ , available in each city. We assume workers' utility  $U_{c,t}$  in city  $c$  at time  $t$  is given by  $U_{c,t} = f(A_{c,t}) H_{c,t}^\alpha Y_{c,t}^{1-\alpha}$ . The workers' problem then becomes

$$\max_{c \in C} U_{c,t} = f(A_{c,t}) H_{c,t}^\alpha Y_{c,t}^{1-\alpha} - r_{c,t} H_{c,t} - Y_{c,t} + \check{w}_{c,t} \quad \text{Eq. 4}$$

<sup>5</sup> This specification of effective population follows Holl (2012), who analysed agglomeration economies in Spain, and has the advantage of avoiding the need to estimate the area of urban areas, which tends to rely on more subjective interpretations and be more prone to measurement error—compared to estimates of resident population.

The worker's budget constraint is defined by  $\check{w}_{c,t} = r_{c,t}H_{c,t} + Y_{c,t}$ , where wages  $\check{w}_{c,t}$  describes the wage earned by residents of city  $c$ , which is distinct from the wages,  $w_{c,t}$ , paid by firms in city  $c$ . The wages  $\check{w}_{c,t}$  earned by workers resident in city  $c$  can, of course, include wages paid to workers who commute to other locations for work. Our data shows, for example, many residents of so-called "satellite towns" earn higher wages by commuting to larger cities nearby.

Using the first-order conditions  $\frac{\partial U_{c,t}}{\partial H_{c,t}} = f(A_{c,t})\alpha H_{c,t}^{\alpha-1} Y_{c,t}^{1-\alpha} - r_{c,t} = 0$  and  $\frac{\partial U_{c,t}}{\partial Y_{c,t}} = f(A_{c,t})(1-\alpha)H_{c,t}^{\alpha} Y_{c,t}^{-\alpha} - 1 = 0$ , we derive the conventional Marshallian demand functions  $H_{c,t} = \alpha \left( \frac{\check{w}_{c,t}}{r_{c,t}} \right)$  and  $Y_{c,t} = (1-\alpha)\check{w}_{c,t}$ . Substituting these two demand functions into Eq. 4 yields the following indirect utility function for a worker located in city  $c$ :

$$v_{c,t} = f(A_{c,t})\alpha^{\alpha}(1-\alpha)^{1-\alpha} \frac{\check{w}_{c,t}}{r_{c,t}^{\alpha}} = V \quad \text{Eq. 5}$$

where  $V$  denotes the so-called "reservation utility" offered by locations outside of our model, which represents the fixed level of utility achieved by all locations in spatial equilibrium.

We substitute the constant  $\kappa = \alpha^{\alpha}(1-\alpha)^{1-\alpha}$  into Eq. 5, and take logs to find  $\log V = \log \kappa + \log f(A_{c,t}) + \log \check{w}_{c,t} - \alpha \log r_{c,t}$ , which can subsequently be re-arranged to yield our "iso-utility equation"  $\alpha \log r_{c,t} = \log \kappa + \log f(A_{c,t}) + \log \check{w}_{c,t} - \log V$ . We model agglomeration economies in consumption using the function  $f(A_{c,t}) = A_c E_{c,t}^{\beta}$ , where  $E_{c,t}$  again denotes the "effective population" as per Eq. 2. The workers' iso-utility equation then becomes:

$$\alpha \log r_{c,t} = \log \kappa + \log A_c + \beta \log E_{c,t} + \log \check{w}_{c,t} - \log V \quad \text{Eq. 6}$$

Eq. 6 suggests rents increase with net consumer amenities,  $\log A_c + \beta \log E_{c,t}$ , and wages,  $\log \check{w}_{c,t}$ , while decreasing with the reservation utility  $\log V$ , which again seems intuitive.

## 2.4 Housing

We propose a simple model of the housing market. First, we assume more cost-effective housing sites are developed first, such that marginal rental costs,  $m_{c,t}$ , increase with city size,  $N_{c,t}$ . As in Glaeser et al (2014), we assume the housing market experiences adjustment frictions, which causes  $m_{c,t}$  to increase with prevailing levels of housing investment,  $I_{c,t}$ . And finally, we assume housing costs increase with resident income,  $\check{w}_{c,t}$ .

Our model of the marginal costs of housing then becomes

$$m_{c,t} = D_c D_t N_{c,t}^{\delta_1} I_{c,t}^{\delta_2} \bar{w}_{c,t}^{\delta_3} \quad \text{Eq. 7}$$

Where  $D_c$  and  $D_t$  denote city and time fixed effects designed to capture the effects of time-invariant local constraints, such as geography and policy, and common (national) trends, respectively. The parameters  $\delta_1, \delta_2, \delta_3 > 0$  govern the elasticity of housing costs to population  $N_{c,t}$ , investment,  $I_{c,t}$ , and resident income,  $\bar{w}_{c,t}$ , respectively.

We assume investment in housing is proportional to growth in the effective population of the region, such that  $I_{c,t} = \frac{E_{c,t}}{E_{c,t-1}}$ . Assuming perfectly competitive housing markets  $r_{c,t} = m_{c,t}$  allows us to derive the following “iso-population” equation that relates rents to housing supply

$$\log r_{c,t} = \delta_1 \log N_{c,t} + \delta_2 \log \left( \frac{E_{c,t}}{E_{c,t-1}} \right) + \delta_3 \log \bar{w}_{c,t} + \bar{D}_c + \bar{D}_t \quad \text{Eq. 8}$$

This model is like that used in Glaeser et. al (2014), with two main differences. First, we use a multiplicative specification for marginal costs. Second, and in contrast to Glaeser et. al (2014), we focus on rental prices, being a flow measure of housing costs. As discussed in Section 3, the New Zealand census collects data on rents, which is consistent with our data on incomes.

## 2.5 Summary

In this section we formulate a simple dynamic version of the Roback model. In our model, equilibrium is defined by three equations linking (1) firm’s demand for labour, (2) worker’s choice of location, and (3) the supply of housing. Adjustment frictions in housing markets are the primary dynamic channel in our model; they link rents to changes in regional housing demand between periods. We include channels for agglomeration economies in both production and consumption, where agglomeration is defined to allow for “spill-overs” from other cities, where the strength of spill-overs attenuates by inter-urban road distance. In this way, agglomeration economies link the economic outcomes of a city to proximate cities. To finish, we note that our model invokes a relatively strong form of spatial equilibrium, which is assumed to hold both *between cities* and *over time*. As noted by Glaser et al (2014), this spatial equilibrium condition implies workers face zero moving costs and have perfect, contemporaneous information on relative prices and amenities in each location. While workers are likely to have reasonable information on prices  $w_{c,t}$  and  $r_{c,t}$ , we question the degree to which they have accurate information on amenity levels. If knowledge of relative amenity levels is less accurate than we assume, then this would seem to introduce more lagged behaviour into the model.

### 3 Data, Estimation, and Results

In this section, we summarise the sources of our data, our approach to estimation, and present results. To finish, we discuss the dynamics implied by our model and results.

#### 3.1 Data

We source much of our data from eight New Zealand Censuses of Population and Dwellings, which span the years from 1976 to 2013.<sup>6</sup> For each urban area, or “city” (defined below), we extract data on incomes and rents from individual records for full time employed individuals aged 25 and over at both their place of residence and place of work.<sup>7</sup> Censuses report annual income and weekly rent values in \$100 and \$10 bands, respectively; we use the midpoint of these bands to estimate averages. We use CPI to adjust all prices to 2013 levels.

Statistics NZ require some values to be suppressed. Counts of 5 or fewer, for example are automatically suppressed, both at the level of totals and sub-groups. After suppression of sensitive values, population counts are randomly rounded to base 3 with 2/3 probability of rounding to the nearest multiple of 3, and 1/3 probability of being rounded to the non-closest multiple of 3. For example, 5 rounds to 6 with 2/3 probability and to 3 with 1/3 probability.<sup>8</sup>

To define a city, we start with the 143 zones defined in the official NZ urban area classification. We consolidate some individual zones into larger metropolitan areas, specifically in Auckland (4 zones), Wellington (4 zones), Hamilton (3 zones), Napier-Hastings (2 zones), and Nelson (2 zones). This leaves us with data on 133 cities spanning eight census, or 1,064 observations in total, which provide our primary units of observation.<sup>9</sup>

#### 3.2 Estimation

To estimate our model, we first define several unobserved variables. For the firm’s iso-cost equation (Eq. 3), we assume local production amenities  $\log P_{c,t} = \bar{P}_c + \bar{\vartheta}_t$ , where  $\bar{P}_c$  and  $\bar{\vartheta}_t$

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<sup>6</sup> The census years are 1976, 1981, 1986, 1991, 1996, 2001, 2006 and 2013.

<sup>7</sup> Access to individual census records is provided by Statistics New Zealand under conditions designed to give effect to the security and confidentiality provisions of the Statistics Act 1975.

<sup>8</sup> A full description of census confidentiality rules is available at <http://archive.stats.govt.nz/Census/2013-census/methodology/confidentiality-how-applied.aspx>

<sup>9</sup> As geographic areas are not consistently coded across censuses, we allocate individuals and dwellings to urban areas as defined in 2013 using the most detailed geographic coding available in each census year. Where a detailed area from an earlier census is associated with more than one urban area in 2013, individual records for the earlier census area is allocated to the urban area that contains the largest share of the 2013 population. Generally, this is a census meshblock, which is a geographic area containing on average around 100 people. For the 1976 census, meshblock codes were derived from undocumented administrative codes. For individuals who were away from home on census night, coding was available only at a more aggregate (area unit) level.

denote city and time fixed effects, respectively. Similarly, for the worker's iso-utility equation (Eq. 6), we assume city-specific amenity levels,  $\log A_c$ , and reservation utility,  $\log V_t$ , are defined by city and time fixed effects,  $\bar{A}_c$  and  $\bar{v}_t$ , respectively. The parameter  $\alpha$  in Eq. 6 defines the cost share of housing in consumption, which is estimated directly from our data  $\alpha = \frac{r_{c,t}}{\tilde{w}_{c,t}} = 0.20$ .<sup>10</sup>

Substituting these definitions into our model yields the following set of equations

$$\log w_{c,t} = \lambda + \frac{1}{1-\gamma} (\epsilon \log E_{c,t} - \gamma \log r_{c,t} + \bar{P}_c + \bar{\vartheta}_t) + \varepsilon_{c,t}^w \quad \text{Eq. 9}$$

$$\log \check{r}_{c,t} = \alpha \log r_{c,t} - \log \tilde{w}_{c,t} = \log \kappa + \beta \log E_{c,t} + \bar{A}_c - \bar{v}_t + \varepsilon_{c,t}^r \quad \text{Eq. 10}$$

$$\log r_{c,t} = \delta_1 \log N_{c,t} + \delta_2 \log \left( \frac{E_{c,t}}{E_{c,t-1}} \right) + \delta_3 \log \tilde{w}_{c,t} + \bar{D}_c + \bar{D}_t + \varepsilon_{c,t}^n \quad \text{Eq. 11}$$

Where we also add error terms  $\varepsilon_{c,t}$  to each equation. We are primarily interested in the following parameters  $\gamma, \epsilon, \beta$ , and  $\delta$ , for which our hypotheses are indicated:

- Cost share of commercial floor space in production,  $\gamma > 0$ ;
- Agglomeration elasticity of production,  $\epsilon > 0$ ;
- Agglomeration elasticity of consumption  $\beta$ ;<sup>11</sup> and
- Housing construction cost parameters  $\delta_1, \delta_2, \delta_3 > 0$  and  $\delta_1 < \delta_2$

Eq. 9 and Eq. 10 can be specified in first-differences, which has the advantage of eliminating time-invariant city-specific fixed effects, namely  $\bar{P}_c$  and  $\bar{A}_c$ , and reducing the number of parameters to be estimated. The downside of this approach is the fixed effects are of economic interest, especially the city-specific amenity levels  $\bar{A}_c$ . For this reason, we chose to estimate Eq. 9 in first-differences as follows

$$\Delta \log w_{c,t} = \frac{1}{1-\gamma} (\epsilon \Delta \log E_{c,t} - \gamma \Delta \log r_{c,t} + \Delta \bar{\vartheta}_t) + \Delta \varepsilon_{c,t}^w \quad \text{Eq. 9b}$$

We estimate the model using three methods. First, we estimate each equation separately using ordinary least squares ("OLS"). While OLS provides a useful benchmark, the estimated

<sup>10</sup> Our estimated value of  $\alpha$  aligns with the values reported in MBIE (2015), as discussed in Maré & Poot (2019).

<sup>11</sup> Tabuchi & Yoshida (2000) find positive consumption amenities in Japan. In contrast, Maré & Poot (2019)—specifically the coefficient on natural log of population—imply consumption disamenities in New Zealand. Due to conflicting evidence on urban consumption amenities, we do not form a hypothesis on the direction of  $\beta$ .

coefficients will be inefficient when disturbances are contemporaneously correlated across equations (Henningsen and Hamann, 2007). As many variables in our equations are related if not identical, we expect shocks will be correlated across equations. In this event, “seemingly unrelated regressions” (SUR) is preferable to OLS. In the presence of endogeneity, however, SUR estimates will be biased. Endogeneity is likely to affect our model in at least two ways: First, lagged dependent variables enter the equations as explanatory variables and, second, endogeneity exists between wages, rents, and agglomeration. To address endogeneity, we estimate the model using two stage least squares (2SLS). For instruments, we follow Anderson and Hsiao (1981), Arrellano and Bond (1991), and Blundell and Bond (1998) and use lagged endogenous variables in levels or differences. We prefer the 2SLS results.

Finally, to estimate standard errors we use block bootstrapping clustered by cities. When estimating the model using SUR and 2SLS, the inclusion of city-specific fixed effects  $\bar{A}_c$  and  $\bar{D}_c$  in Eq. 10 and Eq. 11 caused issues with numerical instability and prevented the model being solved.<sup>12</sup> To address this issue, in the SUR and 2SLS models we replace the city-specific fixed effects with their estimates,  $\hat{A}_c$  and  $\hat{D}_c$ , from the OLS model. In this way, we approximate the fixed effects without estimating them directly within the SUR and 2SLS models. To capture how uncertainty affects our estimates of fixed effects, we adopt a two-step block bootstrapping process: First, we block bootstrap the OLS model to estimate the fixed effects and, second, we use these estimates as explanatory variables into the block bootstrapped SUR and 2SLS models. In the latter two models, the estimated coefficient for the fixed effects represents the degree to which the OLS estimates of  $\hat{A}_c$  and  $\hat{D}_c$  align, on average, with the SUR and 2SLS models.

### 3.3 Main Results

We begin by estimating the system of equations defined by Eq. 9b, Eq. 10, and Eq. 11 for our sample. These benchmark results are summarised in Table 1. For the OLS and SUR models, we have an unbalanced panel consisting of 800 observations for 132 cities over eight census waves from 1981—2013.<sup>13</sup> The sample reduces to 729 observations in the 2SLS model due to the need to use lagged explanatory variables as instruments.

<sup>12</sup> This may be because our panel is relatively wide and short in the cross-sectional and temporal dimensions, respectively. Specifically, we estimate our equations over  $c = 133$  cities over only  $t = 8$  censuses.

<sup>13</sup> Three factors cause our sample to decline the original 1,064 to 800 observations. First, as our iso-cost and iso-utility equations are specified in first-differences, we lose observations associated with the first census wave in 1976. Second, the suppression of data in line with the confidentiality rules discussed in Section 3.1 further reduces the number of observations. Thirdly, we omit observations associated with Waiouru, which is a town linked to an army base of the same name. Economic outcomes in Waiouru seem unlikely to reflect the choices of households and firms.

Table 1: Regression results – Benchmark results

Equation	Term	Models		
		OLS	SUR	2SLS
Iso-cost ( $\Delta \log w_{c,t}$ )	$-\frac{\gamma}{1-\gamma} \Delta \log r_{c,t}$	-0.025 (0.021)	-0.025 (0.020)	0.071 (0.181)
	$\frac{\epsilon}{1-\gamma} \Delta \log E_{c,t}$	0.200 (0.058)***	0.206 (0.055)***	0.190 (0.138)
	Time F.E.	Yes	Yes	Yes
Iso-utility ( $\log \check{r}_{c,t}$ )	$\beta \log E_{c,t}$	-0.155 (0.047)***	-0.148 (0.043)***	-0.121 (0.044)***
	City F.E.	Yes	0.943 (0.009)***	0.959 (0.037)***
	Time F.E.	Yes	Yes	Yes
Iso-population ( $\log r_{c,t}$ )	$\delta_1 \log N_{c,t}$	0.189 (0.061)***	0.174 (0.063)***	0.224 (0.039)***
	$\delta_2 \log \left( \frac{E_{c,t}}{E_{c,t-1}} \right)$	0.800 (0.140)***	1.096 (0.167)***	0.934 (0.169)***
	$\delta_3 \log \check{w}_{c,t}$	0.738 (0.311)**	0.552 (0.262)**	0.556 (0.495)
	City F.E.	Yes	1.019 (0.022)***	1.008 (0.076)***
	Time F.E.	Yes	Yes	Yes
Observations		800	800	729

\*\*\* $p < 0.01$ , \*\* $p < 0.05$ , \* $p < 0.1$ . Block bootstrapping (2003 iterations) used to estimate robust standard errors clustered by city. City fixed effects (F.E.) from the OLS model are used as explanatory variables in the SUR and 2SLS models. F.E. are not reported.

Considering results for our preferred 2SLS model, all parameters have the expected sign with one exception: The cost share of commercial floor space in production,  $\gamma$ , which is found to be negative. This erroneous result may arise due to unobserved differences in industrial composition between cities or measurement error arising from our use of residential rents as a proxy for the price of commercial floor space.

To understand whether this estimate is representative of our sample, we use two statistical measures—specifically Cook’s distance and  $dfbetas$ —to identify and remove influential observations from our data. These observations tend to be associated with small towns dominated by a small number of employers or that were affected by the Christchurch Earthquake, such as Rolleston and Lincoln. Regression results for the sample with influential observations removed are summarised in Table 2. Here, we find the expected positive value for  $\gamma$ . Estimates for other parameters are similar in magnitude and precision to Table 1. For this reason, we prefer results in Table 2, specifically the column associated with 2SLS. Looking at these results, we find relatively large agglomeration economies in production with an elasticity of +0.230, which is significant at the 10% level. The large size of this elasticity may reflect the omission of controls for firm and worker characteristics, which are endogenously determined and positively correlated with agglomeration. In the absence of such controls, our estimate of agglomeration economies in production is likely to be positively biased.



Table 2: Regression results – Sample with influential observations removed

Equation	Term	Models		
		OLS	SUR	2SLS
Iso-cost ( $\Delta \log w_{c,t}$ )	$-\frac{\gamma}{1-\gamma} \Delta \log r_{c,t}$	-0.012 (0.018)	-0.012 (0.019)	-0.089 (0.181)
	$\frac{\epsilon}{1-\gamma} \Delta \log E_{c,t}$	0.189 (0.052)***	0.185 (0.053)***	0.230 (0.132)*
	Time F.E.	Yes	Yes	Yes
Iso-utility ( $\log \check{r}_{c,t}$ )	$\beta \log E_{c,t}$	-0.118 (0.039)***	-0.114 (0.037)***	-0.105 (0.044)**
	City F.E.	Yes	0.951 (0.008)***	0.989 (0.016)***
	Time F.E.	Yes	Yes	Yes
Iso-population ( $\log r_{c,t}$ )	$\delta_1 \log N_{c,t}$	0.282 (0.044)***	0.281 (0.047)***	0.264 (0.042)***
	$\delta_2 \log \left( \frac{E_{c,t}}{E_{c,t-1}} \right)$	0.890 (0.142)***	1.152 (0.160)***	1.039 (0.167)***
	$\delta_3 \log \check{w}_{c,t}$	0.712 (0.225)***	0.551 (0.216)**	0.778 (0.418)*
	City F.E.	Yes	1.027 (0.021)***	0.968 (0.075)***
	Time F.E.	Yes	Yes	Yes
Observations		759	759	696

\*\*\* $p < 0.01$ , \*\* $p < 0.05$ , \* $p < 0.1$ . Block bootstrapping (2003 iterations) used to estimate robust standard errors clustered by city. City fixed effects (F.E.) from the OLS model are used as explanatory variables in the SUR and 2SLS models. F.E. are not reported.

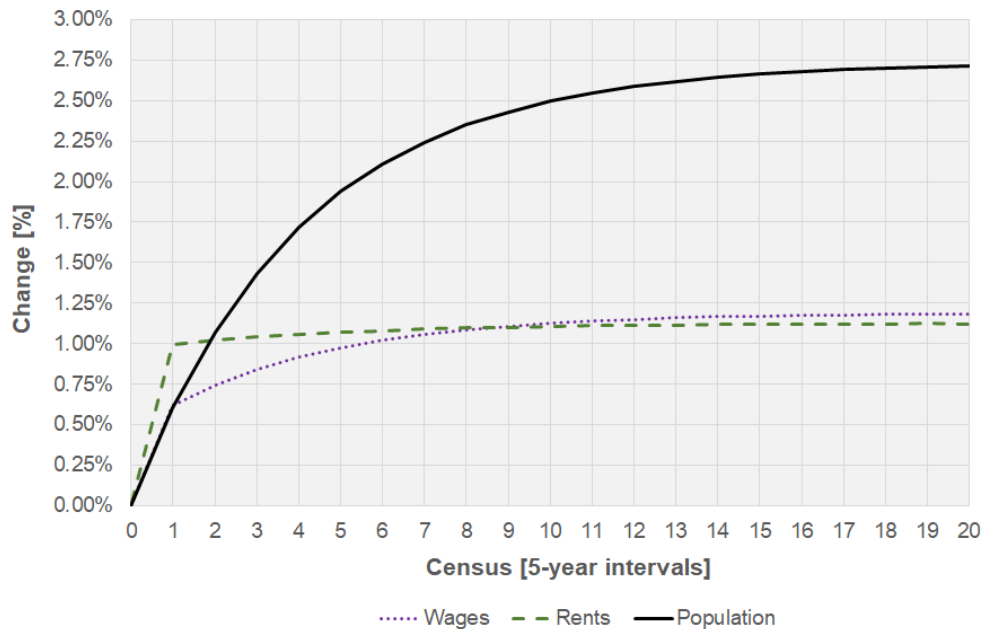
Turning now to agglomeration economies in consumption, we find an elasticity of -0.105, which is significant at the 5% level. As there are few studies of agglomeration economies in consumption in the New Zealand context, we are unable to compare its magnitude. Turning to results for the iso-housing equation, the estimate for  $\delta_1 = 0.264$  is significant at the 1% level and provides evidence that Ricardian notions of comparative advantage apply to urban development, whereby more cost-effective housing sites are developed first. In terms of housing market adjustment frictions, our estimate for  $\delta_2 = 1.039$  is significant at the 1% level and provides evidence housing prices are relatively elastic with respect to changes in the regional population, as measured by effective population density. And the elasticity of housing costs with respect to income is  $\delta_3 = 0.778$ , which is significant at the 10% level. Finally, we note the estimated city F.E. included in the iso-utility and iso-population equations in the SUR and 2SLS models have a coefficient that is close to unity. This suggests the OLS estimates of  $\bar{A}_c$  and  $\bar{D}_c$  are, on average, close to those that would result from the SUR and 2SLS models. In general, estimated coefficients are relatively stable across the three models tested.

Finally, we test the inclusion of additional controls, such as human capital—measured by the percentage of workers with a tertiary education—and trends in agglomeration economies. In all cases, our results were largely unchanged from those reported in Table 2.

### 3.4 Implied Dynamics

Using our estimated parameters, we can simulate the dynamic response predicted by our model. Figure 1 illustrates the effect of a shock to local production advantages denoted by  $\bar{P}_C$ .

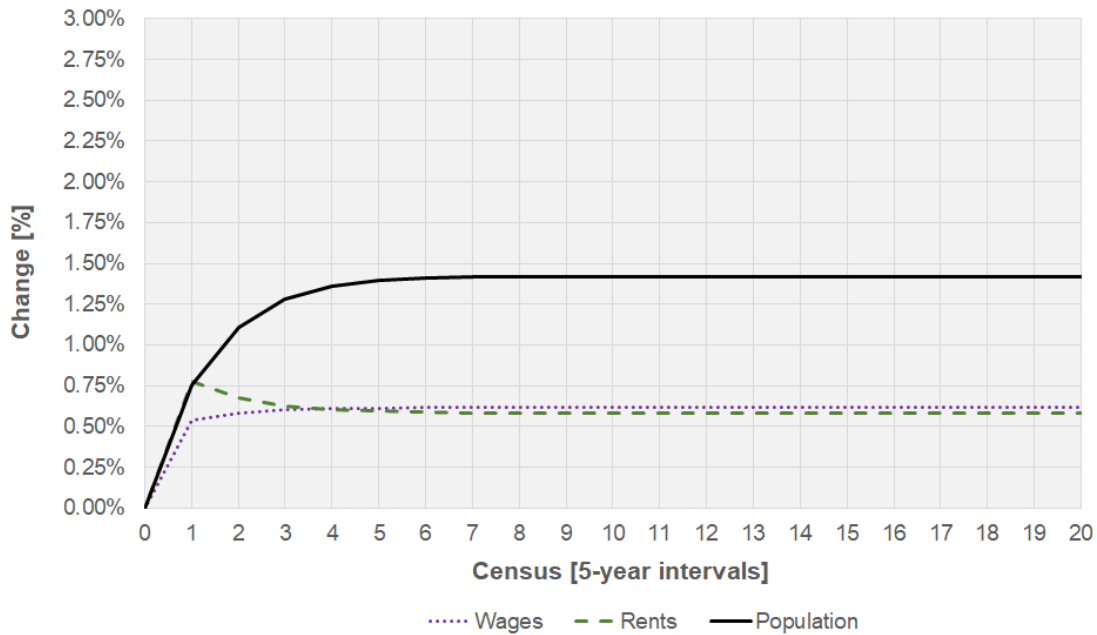
Figure 1: Illustrating the dynamic effects of a productivity shock on wages, rents, and population (measured in natural logs) using estimated model parameters



The productivity shock has the largest immediate impact on rents, which increases sharply before quickly levelling off. Wages also respond relatively quickly yet continue to grow over time as population growth gives rise to additional agglomeration economies in production. The continued, albeit slow, growth in wages indirectly leads to higher rents, due to the income elasticity effect captured by  $\delta_3$ . Compared to the effects on wages and rents, however, the population response is much slower: We find approximately 80 percent of the population response is realised after six censuses, or thirty-years. The speed of the population response tends to reflect two parameters: First, the effects of adjustment frictions in housing markets, as governed by the estimated value for  $\delta_2$  and, second, the effects of agglomeration economies in production, as governed by  $\epsilon$ .

To highlight these effects, we consider an alternative scenario in which we halve the value of  $\delta_2$ , reducing adjustment frictions in housing markets, and also set the value of agglomeration economies in production at  $\epsilon = 0.080$ , which is closer to values reported elsewhere in the literature. Figure 2 shows the effect of these changes on the implied dynamics. The reduction in agglomeration economies leads to a smaller effect on population, of which 80% is realised within circa 10-years of the shock.

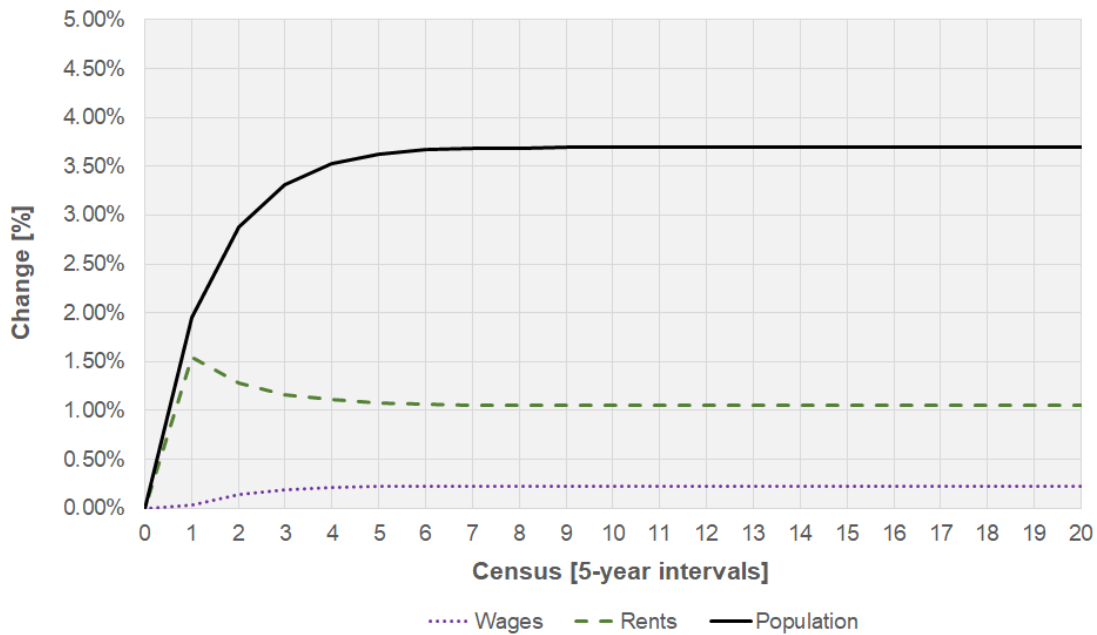
Figure 2: Illustrating the dynamic effects of a productivity shock on wages, rents, and population (measured in natural logs) for alternative values of  $\delta_2$  and  $\epsilon$



We find more muted effects on wages and rents, with the initial spike in the latter tending to subside in later years. Indeed, approximately one-third of the initial spike in rents is found to unwind in subsequent periods. This non-monotonic behaviour in rents reflects the combined effects of adjustment frictions in housing markets and that, in this scenario, agglomeration diseconomies in consumption exceed agglomeration economies in production. Importantly, this non-monotonic behaviour emerges from a relatively simple system of linear equations. In the long run, and for these parameter values, we find the effects of a positive productivity shock are capitalised similarly into wages and rents, that is prices of the two factors of production.

In Figure 3, we consider the dynamics implied by a positive amenity shock using the same parameter values as those used in Figure 2. Here, we see the dynamic response to the positive amenity shock differs in several substantive ways to that found for a productivity shock. Most notably, we observe a larger increase in population that, in turn, leads to a large increase in rents, of which one-third again tends to unwind in later years. This result suggests urban populations are relatively sensitive to relative amenity levels. In terms of wages, we observe a more muted response. Initially, wage growth is largely suppressed by the effects of higher rents. Over time, some of this increase in rent unwinds whereas the population continues to grow, leading to agglomeration economies in production and a small positive increase in wages. For these parameter values, the bulk of the wage response occurs in the period 5-20 years after the initial positive amenity shock. In this scenario, we find the effects of a positive amenity shock on rents are approximately four-times larger than the effect on wages.

Figure 3: Illustrating the dynamic effects of an amenity shock on wages, rents, and population (measured in natural logs) for alternative values of  $\delta_2$  and  $\epsilon$



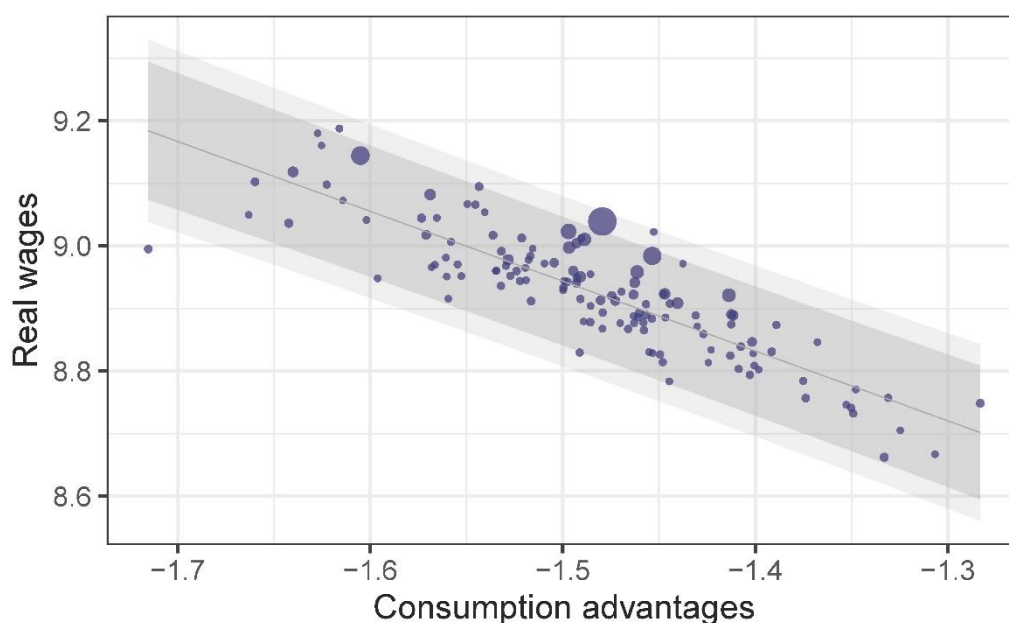
While stylized, these simulations serve three useful purposes. First, we find our model generates relatively plausible predictions about the dynamic effects of productivity shocks on urban wages, rents, and population. Second, we find the combination of adjustment frictions in housing markets and agglomeration economies serve to slow the speed of the population response to shocks. Third, notwithstanding the simplicity of our model, we can generate a relatively wide range of dynamic responses—including some that are non-monotonic—with relatively small changes in parameter values. Fourth, we find productivity and amenity shocks have relatively distinct effects on economic outcomes: Whereas the former is capitalised approximately equally into wages and rents, the latter tends to be capitalised more into rents. And finally, these scenarios highlight the usefulness of considering a spatial general equilibrium, which explicitly models economic interactions between firm’s demand for labour, household’s location choices, and the functioning of housing markets.

## 4 Discussion

### 4.1 Spatial Equilibrium

First, we assess the assumption of spatial equilibrium. In the worker’s iso-utility equation, the terms  $\bar{A}_c + \beta \log E_{c,t}$  measure the value of city-specific amenities and agglomeration economies in consumption, respectively, which we describe as “consumption advantages”. In contrast, the term  $\log \check{w}_{c,t} - \alpha \log r_{c,t}$  measures the value of real wages. Figure 4 plots consumption amenity (horizontal axis) versus real wages (vertical axis) for each city in 2013.

Figure 4: Comparing real wages versus consumption advantages (Note: The light and dark shaded areas illustrate the 99<sup>th</sup> and 95<sup>th</sup> percentile for the prediction intervals of the regression, respectively)



We find a strong negative relationship, where real wages and consumption amenities effectively compensate each other. Indeed, if all locations are to achieve the same reservation utility level,  $\bar{v}_t$ , then real wages *must* compensate for consumption amenities and vice versa. Put another way, regressing real wages with consumption advantages should return a slope of -1. Estimating this regression using our estimated parameter estimates and 2013 data, we find a mid-point estimate for the slope of -1.116 (CI -1.276, -0.9555). As the confidence interval of the slope includes -1, we cannot reject the assumption that spatial equilibrium holds for these cities and towns in 2013 and for these parameter estimates. Notably, all cities except for Kawerau lie within the 99% prediction interval of this regression.

Second, we undertake an analogous analysis of spatial equilibrium for firms. In this analysis, we compare city-specific elements of the firm’s iso-cost equation. Specifically, we

compare the total unit costs that firms incur, defined by  $(1 - \gamma) \log w_{c,t} + \gamma \log r_{c,t}$ , with the production advantages defined by  $\bar{P}_c + \epsilon \log E_{c,t}$ . Figure 5 plots productive advantages (horizontal axis) versus total costs (vertical axis) for each city, again in 2013.

Figure 5: Comparing total costs versus production advantages (Note: The light and dark shaded areas illustrate the 99<sup>th</sup> and 95<sup>th</sup> percentile prediction intervals of the regression, respectively)

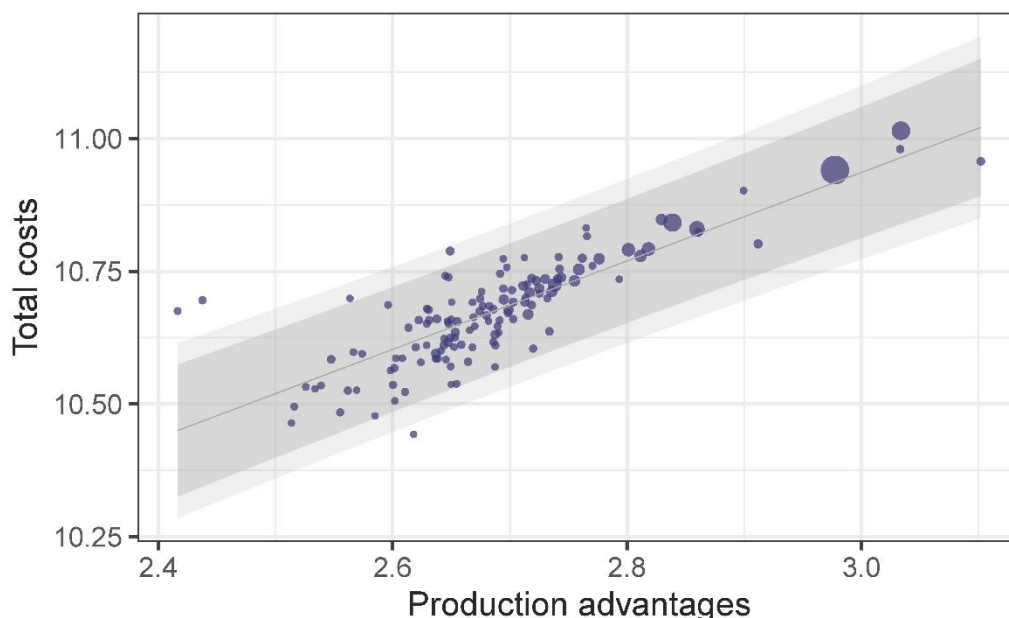


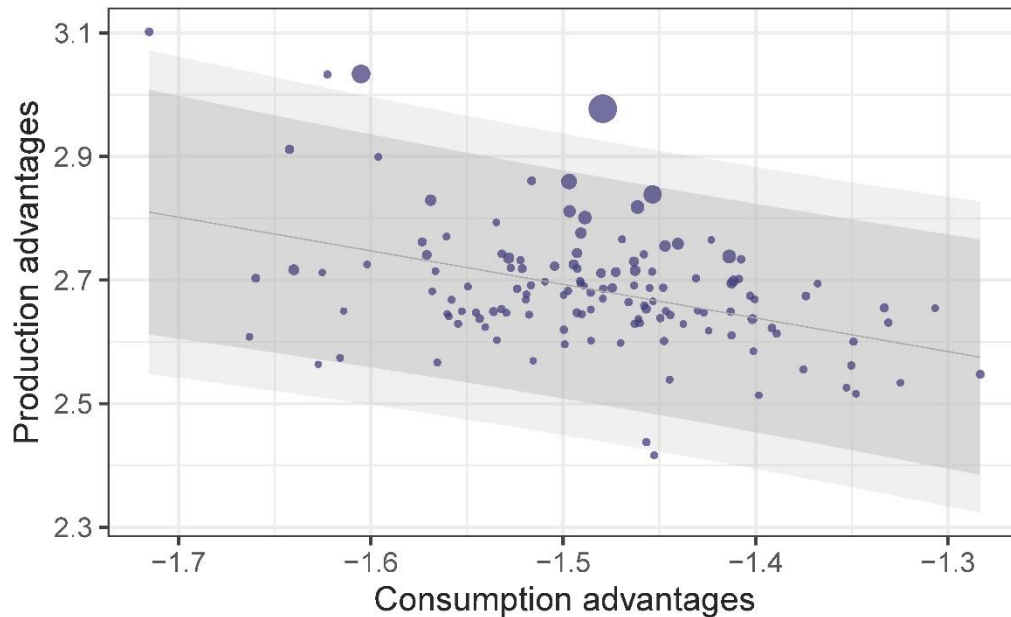
Figure 5 reveals a strong positive association between total costs and production advantages at the city level in 2013: Relative differences in local productive advantages appear to be arbitrated away by local prices for labour and floor space. For spatial equilibrium to hold for firms, we expect to find a positive slope of +1. Regressing total costs versus production advantages returns a mid-point estimate for the slope of 0.833 (CI 0.6848, 0.982), which excludes +1 at the 95% level. In this case, we observe two smaller towns with relatively high costs and low production advantages, namely Mapua and Snells Beach. If we re-run the regression excluding these two towns, then the slope of the regression increases to 0.930 which includes 1.000 at the 95% level of confidence.

Based on these results, we find little evidence to reject the assumption of spatial equilibrium for New Zealand’s cities and towns in 2013, at least on average. Perhaps the most notable systematic departure from a deterministic interpretation of spatial equilibrium occurs for New Zealand’s larger cities, which appear to offer households relatively attractive bundles of real wages and consumption advantages. This can be seen in Figure 4, where larger cities are all located above the regression line that defines the “average” bundles of real wages and consumption advantages offered by cities and towns across New Zealand.

## 4.2 Producer vis-à-vis Consumer Cities

Recent research has emphasised the twin roles of cities as places of production and consumption (see, for example, Glaeser and Gottlieb, 2006). We can use our results to estimate the relative value placed on productive and consumer advantages in each city, defined by  $\bar{P}_c + \epsilon \log E_{c,t}$  and  $\bar{A}_c + \beta \log E_{c,t}$ , respectively. These advantages are illustrated in Figure 6.

Figure 6: Production versus consumption advantages (Note: The light and dark shaded areas illustrate the 99<sup>th</sup> and 95<sup>th</sup> percentile prediction intervals of the regression, respectively)



Here we find a slight negative association, which implies locations with higher production advantages tend to have, on average, lower consumption advantages. In Figure 6, we can see how two of New Zealand's larger cities, Auckland and Wellington, offer combinations of production and consumption advantages that are somewhat uniquely attractive.

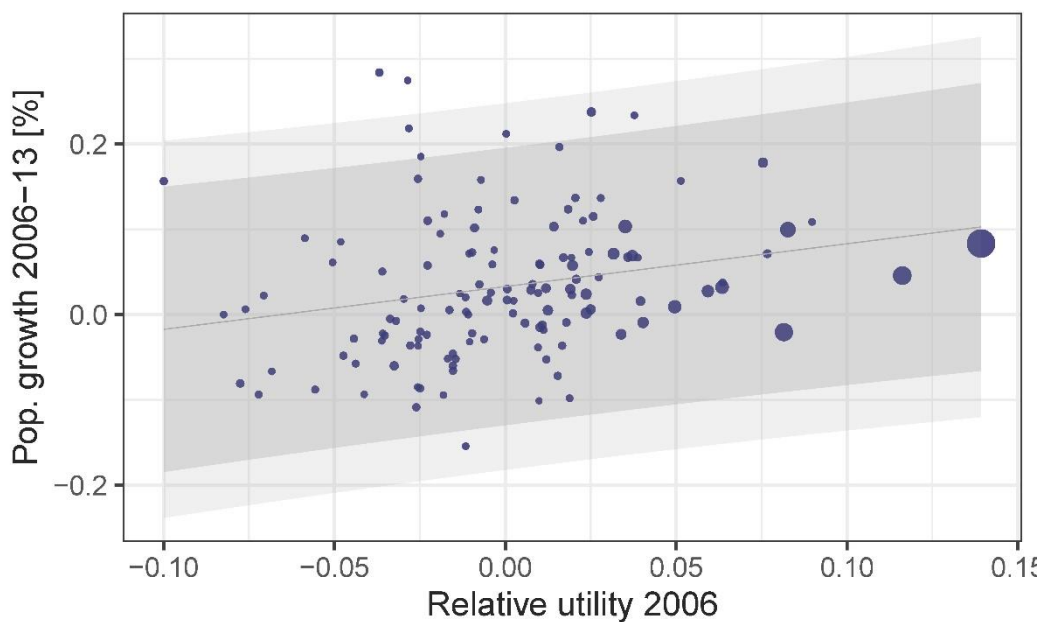
## 4.3 Departures from Spatial Equilibrium

In Section 4.1, we find no strong evidence to reject the assumption of spatial equilibrium in 2013. At the same time, we do observe small variations in relative utility between cities. We posit these variations represent small localised departures from spatial equilibrium caused by the lingering effects of historical shocks to relative productivities and / or amenities. As we saw in Section 3.4, adjustment frictions in housing markets and agglomeration economies can slow the speed with which a long-run equilibrium is reached after a shock. In this way, historical shocks may give rise to persistent localised differences in relative utility levels between cities. This raises two interesting empirical questions. First, can we analyse whether population

growth responds to departures from spatial equilibrium and, second, can we assess whether New Zealand's cities and towns are closer to spatial equilibrium in 2013 than they were 2006?

To answer the first question, Figure 7 plots population growth in the period from 2006—2013 on the vertical axis versus relative utility in 2006 on the horizontal axis, both mean centred.

Figure 7: Population growth 2006-13 [%] versus relative utility levels in 2006<sup>14</sup>



We find a positive association between relative utility in 2006 and subsequent population growth, which is statistically significant ( $p$ -value  $< 0.05$ ). This positive and significant effect is preserved even when we include historical population growth as an additional regressor with differences in relative utility. This finding suggests urban population growth does indeed respond to localised departures from spatial equilibrium, as we would expect.<sup>15</sup>

To answer the second question on whether localised departures from spatial equilibrium are increasing or reducing over time, we compare relative utility in 2013 to that in 2006. We find a strong positive association (correlation 0.641), confirming differences in relative utility persist over time, as implied by our results. When we compare relative utility levels in 2006 with 2013, however, we find the latter has a larger spread of outcomes. An increase in the spread of relative utility levels seems to imply New Zealand's cities and towns are, on average, further from a common deterministic spatial equilibrium in 2013 than they were in 2006.

<sup>14</sup> Here, we exclude the towns of Rolleston and Lincoln from the analysis. The population growth of these towns was significantly affected by the Christchurch Earthquake.

<sup>15</sup> One implication of this finding is that departures from spatial equilibrium, as estimated using our model, could be incorporated into the models used to develop population projections, such as those applied by Statistics New Zealand.



This interesting finding raises a related question: Why are New Zealand’s cities and towns further from spatial equilibrium in 2013 than 2006? We can see two potential explanations.

The first explanation is urban population growth experiences positive feedback, such that localised departures from spatial equilibrium give rise to changes in population that subsequently cause even larger departures. This outcome is analogous to the so-called “black hole condition” in some spatial economic models, where—for certain parameter values—the models predict all economic activity will end up in one location, that is, complete agglomeration. Our analysis of implied dynamics in Section 3.4 suggests the black hole condition does not apply for our estimated parameter values. This can be seen by the fact that the predicted population response to the shock gradually dampens out over time, as opposed to being amplified.

The second potential explanation as to why New Zealand’s cities and towns are further from spatial equilibrium in 2013 than they were in 2006 is simply that the intervening period saw shocks that were too large to be dampened via population movements in the time available between censuses. Against a volatile background formed by the 2007 global financial crisis (GFC), the rapid increase in New Zealand’s net migration from 2010 onwards, and the 2011 Christchurch Earthquake, we find this explanation to be rather convincing. In addition to these large shocks, the potential exists for additional local shocks arising from changes in technologies and policies that may serve to change the relative attractiveness of locations. For these reasons, we suggest—but cannot prove—New Zealand’s cities may be further from spatial equilibrium in 2013 than in 2006 simply due to the magnitude of shocks in the intervening period.

We also analysed trends in spatial equilibrium for firms and reached similar findings to those for households. That is, we observe strong persistence in relative production advantages between 2006 and 2013 (correlation +0.609) as well as increased departures from spatial equilibrium in 2013 compared to 2006, as measured by the standard deviation of production advantages between cities in each year.

#### 4.4 The Role of Natural Amenities

To finish, we consider the role of natural amenities. In our model, our city fixed effects  $\bar{A}_c$  capture *exogeneous amenities*, such as climate, as well as persistent *endogenous amenities*, such as durable public facilities and other social infrastructure. We are interested in the potential contribution of natural amenities to these city fixed effects. To answer this question, we regress the city fixed effects against a set of climatic, geographic, and recreational variables. Results are presented in Table 3. All variables have the expected sign and several are statistically significant ( $p < 0.05$ ), specifically rain, temperature, coastal locations, and the share of the workforce

employed in recreational activities. Together, these variables explain approximately one-quarter of the variation in city-specific fixed effects, which is significant.

Table 3: Explaining the contribution of natural amenities to city fixed effects  $\bar{A}_c$

Term	Estimate	Std Error	t-statistic	p-value
Rain	-0.001	0.000	-2.58	0.011
log(Temp)	0.389	0.093	4.18	0.000
Wind	-0.002	0.003	-0.64	0.525
Sun	0.001	0.001	0.88	0.380
$D_{coastal}$	0.038	0.021	1.82	0.071
$S_{rec}$	0.025	0.008	3.38	0.001

132 observations. Robust standard errors.

Using these results, we can estimate the relative value of natural amenities for individual cities and towns. Table 4 lists cities and towns in New Zealand with the ten highest and lowest natural amenities, as per variables in Table 3. All top ten spots are occupied by cities and towns in the North Island, with seven of the ten lowest spots occupied by towns in the South Island.

Table 4: Ranking natural amenities in New Zealand cities and towns – Ten highest and lowest

Ten highest ...		Ten lowest ...	
Rank	City / Town	Rank	City / Town
1	Paihia	123	Milton
2	Waiheke Island	124	Raetihi
3	Ngunguru	125	Winton
4	Auckland	126	Gore
5	Tauranga	127	Reefton
6	Gisborne	128	Balclutha
7	Whakatane	129	Pleasant Point
8	Napier-Hastings	130	Bluff
9	Whangarei	131	Tokoroa
10	Thames	132	Eltham

Of course, important aspects of amenity are not captured in these variables. Looking at the model residuals, we find the largest residuals exist for larger cities, such as Auckland, Christchurch, Hamilton, Dunedin, and Wellington, as well as smaller towns that offer unique amenities, such as Rotorua. Unexplained variation for larger cities may reflect the value of durable public facilities and other social infrastructure, such as schools, universities, libraries, and parks, which are not explicitly included in the model.

## 5 Conclusions

In this working paper, we set out to model urban development in New Zealand. Our main contribution is to formulate and estimate an exceptionally simple dynamic spatial general equilibrium model that links labour and housing markets to workers' location choice. Whereas most existing research considers long-run cross-sectional outcomes between locations, we extend the Roback model to consider outcomes between locations and over time.

Estimating our model using data for 132 urban areas in New Zealand for the period 1976–2013 generates parameter estimates that are consistent with expectations and, in most cases, statistically significant. Notably, we find evidence of agglomeration economies in production and diseconomies in consumption. While larger cities offer firms productive advantages, these are—to some extent—offset by disadvantages for households. Our estimates for agglomeration economies in production are larger than those found in other studies, which likely reflects the absence of controls for firm and worker characteristics that are both positively correlated with wages and endogenously determined with agglomeration. In terms of housing, we find costs increase with city size. This suggests urban housing markets are characterised by Ricardian notions of comparative advantage, where the most cost-effective sites are developed first. Importantly, we also find evidence of adjustment frictions in housing markets, where increased regional demand gives rise to elevated local housing costs. And, finally, our estimate of the elasticity of housing costs with respect to income is positive and significant, as expected.

Drawing on our theoretical model and empirical results, we then explore the changes to dynamic spatial equilibrium in the New Zealand's context. In the wake of a positive productivity shock, for example, we find rents and wages adjust more quickly than the population. The speed of the latter appears to be primarily governed by two factors: Adjustment frictions in housing markets and agglomeration economies, both of which slow the speed of population adjustment. Depending on parameter values, our results suggest 80 percent of the population adjustment to a productivity shock is realised after 10-30 years. These are relatively long timeframes. We also consider the effect of an amenity shock, which we find is largely capitalised into rents.

Importantly, we do not find evidence to reject the assumption of spatial equilibrium 2013 for households or firms. We also consider the dual role of cities as places of production and consumption and, on average, find a negative association between these two outcomes in the New Zealand context: Cities that are advantageous for firms tend to be less advantageous for households, and vice versa. In terms of localised departures from spatial equilibrium, differences in the value that households attach to locations in 2006, as estimated by our model,

is found to be a positive and significant predictor of subsequent population growth in the period 2006-13. While population adjustments serve to dampen out localised departures from spatial equilibrium, the size of these differences appeared to increase in 2013 vis-à-vis 2006. We expect the latter result may reflect the lingering effects of large shocks in the intervening period, such as the GFC and the Christchurch Earthquake. Finally, we relate our city fixed effects—which captures attractiveness that remains constant over time—to aspects of natural amenities, such as climate, geography, and recreation. Results suggest these natural amenities explain approximately one-quarter of the estimated city fixed effects. While this is significant, it leaves considerable scope for policy settings to influence the attractiveness of locations.

Our findings provide fertile ground for further research. Here, we outline several ways in which our analysis could be extended.

First, our theoretical model is simplistic to the point of being mechanistic. Given this working paper represents the model's first outing, we feel comfortable with this choice. That said, further work could seek to extend its theoretical foundations in ways that are relevant for applied work. In the appendix, for example, we show how the model is readily extended to incorporate heterogeneity in worker's productivities and preferences.

Second, the time dimension of our data is relatively short. This is exacerbated by the need to estimate some equations in differences and use lagged variables as instruments in some models. In the New Zealand context, only the passage of time and the inclusion of additional census waves can address this issue. Fortunately, results for the 2018 census should be available shortly. More extensive and detailed panel data sets may exist that are better suited to estimating such models.

Third, in addition to adjustment frictions in housing markets, other potential market frictions may be relevant to explaining the dynamics of urban development. Labour search costs, information barriers, and relocation costs may also play a role, as might the functioning of capital markets. To model the latter, one could introduce capital as an input into production where the price is affected by aggregate national investment in housing. In this way, the cost of capital inputs into the production of goods and housing will be connected to development levels nationally, giving rise to the potential for additional market interactions.

Fourth, we could seek to capture differences in industrial composition that may give rise to unique trends and shocks.<sup>16</sup> Modelling sectoral differences may provide additional explanations for observed compositional and temporal differences between urban settlements. Such work could consider more flexible production functions that, for example, modelled agglomeration economies in a non- Hicks neutral manner. Such analysis would require comparable sectoral classifications over time.

Fifth, while our model is dynamic, all its parameters are static. This implies production processes for tradeable goods, people's preferences over locations, the nature of agglomeration processes, and the functioning of housing markets do not change in the three to four decades that we analyse. With more temporal data, it might be possible to relax the assumption of static parameters to identify trends in these economic processes.

Finally, at present we treat cities and towns as points in space, albeit points that are related by inter-urban agglomeration economies. We do not, however, model intra-urban economic outcomes, such as transport costs. Intuitively, we would expect transport costs to rise with city size, especially at the point that road network congestion becomes commonplace. Further work could seek to explicitly model intra-urban outcomes in a way that integrated with the inter-urban processes in the present model, such as agglomeration economies.

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<sup>16</sup> Coleman, Maré, & Zheng (2019) document changes in the sectoral composition of New Zealand's workforce in the period covered by our data that would seem to favour larger cities, such as Auckland. At the same time, they document a trend towards urban workforces becoming more diversified and less reliant on specialised industries.

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# Appendix



## Appendix A – Heterogeneous worker specification

### Preliminaries

Here, we extend our model to multiple worker types. Our motivation for relaxing the assumption of homogeneous workers reflects empirical evidence on compositional differences between cities, which suggests—among other things—that the returns to agglomeration vary with skill (Bacolod, Blum, & Strange, 2009; Glaeser & Maré, 2001). Such findings explain, for example, the tendency for skilled workers to concentrate in large cities. The risk exists that differences in skills are correlated with location choice and, in turn, bias parameters estimated from the single worker model. Again we stand on the shoulders of giants: Roback (1988) extends her earlier model to include heterogeneous worker types, which enter firms' production functions as imperfect substitutes.<sup>17</sup> To solve the model, Roback (1988) invokes spatial equilibrium between cities within worker types. That is, spatial equilibrium leaves workers of the same type indifferent between locations. Spatial equilibrium does not imply, however, indifference between worker types: high skilled workers receive higher wages and attain higher utility than low-skilled workers. In the following sub-sections, we extend our simple model to accommodate two worker types in the spirit of Roback (1988). We allow for heterogeneity in two dimensions, namely productivity and preferences, and categorise workers as high and low skills based on whether they have tertiary qualifications.<sup>18</sup>

### Firms

We follow Section 2.2, although firms can now employ a mix of high (h) and low (l) skilled workers,  $N_c^h$  and  $N_c^l$ , which are paid distinct wages,  $w_c^h$  and  $w_c^l$ . We assume both worker types incur the same costs of floor space,  $\mu r_c$ . Formally, firms in each city seek to maximise profit  $\pi_c$

$$\pi_{c,t} = g(P_{c,t}) L_{c,t}^\gamma N_{c,t}^h{}^\eta N_{c,t}^l{}^{1-\gamma-\eta} - \mu r_{c,t} L_{c,t} - w_{c,t}^h N_{c,t}^h - w_{c,t}^l N_{c,t}^l \quad \text{Eq. 12}$$

From which we can derive three first-order conditions

$$\begin{aligned} \frac{\partial \pi_{c,t}}{\partial L_{c,t}} &= g(P_{c,t}) \gamma L_{c,t}^{\gamma-1} N_{c,t}^h{}^\eta N_{c,t}^l{}^{1-\gamma-\eta} - \mu r_{c,t} = 0 \\ \frac{\partial \pi_{c,t}}{\partial N_{c,t}^h} &= g(P_{c,t}) \eta L_{c,t}^\gamma N_{c,t}^{h-1} N_{c,t}^l{}^{1-\gamma-\eta} - w_{c,t}^h = 0 \end{aligned}$$

<sup>17</sup> Hence, firms demand strictly positive quantities of all worker types in equilibrium, excluding the potential for unrealistic corner, or "ghetto", solutions where the entire population of individual cities consist of one worker type.

<sup>18</sup> Here, we assume worker type is observed by firms and exogenous to location choice. The latter seems reasonable given decisions on education are usually taken *before* people are counted in the census as full-time workers.

$$\frac{\partial \pi_{c,t}}{\partial N_{c,t}^l} = g(P_{c,t})(1 - \gamma - \eta)L_{c,t}^\gamma N_{c,t}^h{}^\eta N_{c,t}^l{}^{\gamma-\eta} - w_{c,t}^l = 0$$

From these FOCs we can derive expressions for  $L_{c,t}$  and  $N_{c,t}^l$  in terms of  $N_{c,t}^h$  and prices  $r_{c,t}$ ,  $w_{c,t}^h$ , and  $w_{c,t}^l$ . Imposing a zero-profit condition and solving for  $w_{c,t}^h$  yields our wage, or iso-cost, equation for high-skilled workers

$$w_{c,t}^h = g(P_{c,t})^{\frac{1}{\eta}} \left( \frac{1}{\mu r_{c,t}} \right)^{\frac{\gamma}{\eta}} \left( \frac{1}{w_{c,t}^l} \right)^{\frac{1-\gamma-\eta}{\eta}} \eta^{\frac{1}{\eta}} \left( \frac{\gamma}{\eta} \right)^{\frac{\gamma}{\eta}} \left( \frac{1-\gamma-\eta}{\eta} \right)^{\frac{1-\gamma-\eta}{\eta}}$$

Or in logs

$$\log w_{c,t}^h = \lambda^h - \frac{\gamma}{\eta} \log r_{c,t} - \frac{1-\gamma-\eta}{\eta} \log w_{c,t}^l + \frac{1}{\eta} \log g(P_{c,t}) \quad \text{Eq. 13}$$

Where  $\lambda^h = \log \left[ \eta^{\frac{1}{\eta}} \left( \frac{\gamma}{\eta} \right)^{\frac{\gamma}{\eta}} \left( \frac{1-\gamma-\eta}{\eta} \right)^{\frac{1-\gamma-\eta}{\eta}} \right] - \frac{\gamma}{\eta} \log \mu$ . Analogously, we derive the following iso-cost equation for the wages of low-skilled workers  $w_c^l$

$$\log w_{c,t}^l = \lambda^l - \frac{\gamma}{1-\gamma-\eta} \log r_{c,t} - \frac{\eta}{1-\gamma-\eta} \log w_{c,t}^h + \frac{1}{1-\gamma-\eta} \log g(P_{c,t}) \quad \text{Eq. 14}$$

$$\text{Where } \lambda^l = \log \left[ (1-\gamma-\eta)^{\frac{1}{1-\gamma-\eta}} \left( \frac{\gamma}{\eta} \right)^{\frac{\gamma}{\eta}} \left( \frac{\eta}{1-\gamma-\eta} \right)^{\frac{\eta}{1-\gamma-\eta}} \right] - \frac{\gamma}{1-\gamma-\eta} \log \mu.$$

## Workers

Mobile workers choose the city that maximises their utility, where  $U_{c,t}^j$  denotes the utility of worker type  $j$  in city  $c$  at time  $t$ . We follow the approach in Section 2.3, with one extension: We assume rents for each worker type are a fixed multiple  $\zeta^j$  of the mean rent observed in city,  $r_c$  where  $\zeta^h > \zeta^l$ . The effect of  $\zeta^j$  is to shift the price paid per dwelling for each worker type in response to unobserved quality attributes. Workers' utility  $U_{c,t}^j$  in city  $c$  is again given by  $U_{c,t}^j = f^j(A_{c,t}^j) H_{c,t}^\alpha Y_{c,t}^{1-\alpha}$ , such that the workers' optimisation problem becomes  $\max U_{c,t}^j = f(A_{c,t}^j) H_{c,t}^\alpha Y_{c,t}^{1-\alpha} - \zeta^j r_{c,t} H_{c,t}^j - Y_{c,t}^j + \tilde{w}_{c,t}^j$ , from which we derive the following first-order conditions

$$\frac{\partial U_{c,t}^j}{\partial H_{c,t}^j} = f(A_{c,t}^j) \alpha H_{c,t}^{\alpha-1} Y_{c,t}^{1-\alpha} - \zeta^j r_{c,t} = 0 \quad \forall j$$

$$\frac{\partial U_{c,t}^j}{\partial Y_{c,t}^j} = f(A_{c,t}^j) (1-\alpha) H_{c,t}^\alpha Y_{c,t}^{-\alpha} - 1 = 0 \quad \forall j$$

And the associated Marshallian demand functions  $H_{c,t}^j = \alpha \left( \frac{\tilde{w}_{c,t}^j}{\zeta^j r_{c,t}} \right)$  and  $Y_{c,t}^j = (1 - \alpha) \tilde{w}_{c,t}^j$ , which yields the following indirect utility in spatial equilibrium for each worker type

$$v_c^j = f(A_{c,t}^j) \alpha^\alpha (1 - \alpha)^{1-\alpha} \frac{\tilde{w}_{c,t}^j}{(\zeta^j r_{c,t})^\alpha} = V^j \quad \forall j \quad \text{Eq. 15}$$

Where  $V^j$  again denotes the reservation utility that worker type  $j$  attaches to non-urban locations, for example rural areas or, alternatively, overseas.

Again, we assume agglomeration economies in consumption observe  $f^j(A_{c,t}^j) = A_c^j E_{c,t}^{\beta^j}$ , where preferences for exogeneous amenities,  $A_c^j$ , and agglomeration economies,  $\beta^j$ , vary by worker type. Substituting the expression for consumer amenities into Eq. 15, taking logs, and letting  $\kappa = \alpha^\alpha (1 - \alpha)^{1-\alpha}$  yields  $\log V^j = \log \kappa + \log A_c^j + \beta^j \log E_{c,t} + \log \tilde{w}_{c,t}^j - \alpha \log(\zeta^j r_{c,t})$ . Re-arranging yields the iso-utility equations

$$\log \check{r}_{c,t}^j = \alpha \log r_{c,t} - \log \tilde{w}_{c,t}^j = \log \kappa + \log A_c^j + \beta^j \log E_{c,t} - \log V^j \quad \forall j \quad \text{Eq. 16}$$

### Housing

We treat housing as per the single worker case, except we now weight the population for each worker type by  $\zeta^j$  to capture the effects of workforce composition on the marginal cost of housing. Formally, our expression for the marginal cost of housing becomes

$$m_{c,t} = D_c D_t (\zeta^h N_{c,t}^h + \zeta^l N_{c,t}^l)^{\delta_1} \left( \frac{E_{c,t}}{E_{c,t-1}} \right)^{\delta_2} \tilde{w}_{c,t}^{\delta_3} \quad \text{Eq. 17}$$

If  $\zeta^j = \zeta = 1$ , then Eq. 17 collapses to the single worker case. Following the same approach as Section 2.4 leads to the following population equation:

$$\log r_{c,t} = \delta_1 \log \hat{N}_{c,t} + \delta_2 \log \left( \frac{E_{c,t}}{E_{c,t-1}} \right) + \delta_3 \log \tilde{w}_{c,t} + \bar{D}_c + \bar{D}_t \quad \text{Eq. 18}$$

The term  $\hat{N}_{c,t} = \zeta^h N_{c,t}^h + \zeta^l N_{c,t}^l$  denote total housing units demanded in each city and time period, adjusted for quality. The implication of this specification is demand from high-skilled workers will have a larger effect on marginal prices than low-skilled workers, due to differences in the quality of housing demanded.

## Summary

Pulling together the components of the model yields the following five equations:

$$\Delta \log w_{c,t}^h = \frac{1}{\eta} \left[ -\gamma \Delta \log r_{c,t} - (1 - \gamma - \eta) \Delta \log w_{c,t}^l + \epsilon \Delta \log E_{c,t} + \bar{\vartheta}_t \right] + \varepsilon_{c,t}^{w^h}$$

$$\Delta \log w_{c,t}^l = \frac{1}{\eta} \left[ -\gamma \Delta \log r_{c,t} - (1 - \gamma - \eta) \Delta \log w_{c,t}^h + \epsilon \Delta \log E_{c,t} + \bar{\vartheta}_t \right] + \varepsilon_{c,t}^{w^l}$$

$$\log \check{r}_{c,t}^h = \alpha \log r_{c,t} - \log \check{w}_{c,t}^h = \log \kappa + \log A_c^h + \beta^h \log E_{c,t} - \log V^h + \varepsilon_{c,t}^{r^h}$$

$$\log \check{r}_{c,t}^l = \alpha \log r_{c,t} - \log \check{w}_{c,t}^l = \log \kappa + \log A_c^l + \beta^l \log E_{c,t} - \log V^l + \varepsilon_{c,t}^{r^l}$$

$$\log r_{c,t} = \delta_1 \log \hat{N}_{c,t} + \delta_2 \log \left( \frac{E_{c,t}}{E_{c,t-1}} \right) + \delta_3 \log \check{w}_{c,t} + \bar{D}_c + \bar{D}_t + \varepsilon_{c,t}^n$$

As noted earlier, we can estimate  $\zeta^h$  and  $\zeta^l$  from our data and use the resulting values to calculate composition adjusted housing demand,  $\hat{N}_{c,t} = \zeta^h N_{c,t}^h + \zeta^l N_{c,t}^l$ . We note the number of equations can be reduced further by subtracting one iso-utility equation from the other, to yield:

$$\log \left( \frac{\check{r}_{c,t}^h}{\check{r}_{c,t}^l} \right) = \log \left( \frac{A_c^h}{A_c^l} \right) + (\beta^h - \beta^l) \log E_{c,t} - \log \left( \frac{V^h}{V^l} \right) + (\varepsilon_{c,t}^{r^h} - \varepsilon_{c,t}^{r^l})$$

Under this specification, the coefficients provide information on the statistical significance of the distinction between worker types. And ultimately this leaves us with four equations to estimate, which is only one more than the number of equations in the single worker case.